Chapter 3

Prob. 3.1

Calculate Bohr radius for donor in Si ($m_n^* = 0.26\, m_0$).

From Eq.(2-10) with $n = 1$ and using $\varepsilon_r = 11.8$ for Si:

$$r = \frac{4\pi\varepsilon_r\varepsilon_0\hbar^2}{m_n^* q^2} = \frac{11.8(8.85\times10^{-12})(6.63\times10^{-34})^2}{\pi(0.26)(9.11\times10^{-31})(1.6\times10^{-19})^2}$$

$$r = 2.41\times10^{-9}\text{m} = \mathbf{24.1\text{Å}}$$

Note that this is more than four lattice spacings a (5.43Å)

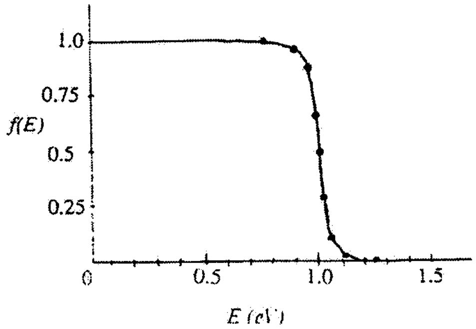
Prob. 3.2

Plot Fermi function for $E_F = 1$ eV.

$$f(E) = [1 + e^{(E-E_F)/kT}]^{-1}$$

We will choose E in eV and therefore use $kT = 0.0259$

E(eV)	$(E-E_F)/kT$	$f(E)$
0.75	-9.6525	**0.99994**
0.90	-3.8610	**0.97939**
0.95	-1.9305	**0.87330**
0.98	-0.7722	**0.68399**
1.02	+0.7722	**0.31600**
1.05	+1.9305	**0.12669**
1.10	+3.8610	**0.02061**
1.25	+9.6525	**0.00006**



Prob. 3.3

Calculate the density-of-states effective mass associated with the X minimum for the given band structure.

Given that near the energy minimum along [1 0 0], the band structure is:

$$E = E_0 - A\cos(\alpha k_x) - B\{\cos(\beta k_y) + \cos(\beta k_z)\}$$

which can be Taylor expanded near the minima:

$$E \approx E_0 - A\left[1 - 2\left(\frac{\alpha k_x}{2}\right)^2\right] - B\left[2 - 2\left(\frac{\beta k_y}{2}\right)^2 - 2\left(\frac{\beta k_z}{2}\right)^2\right]$$

$$\approx (E_0 - A - 2B) + \frac{A}{2}\alpha^2 k_x^2 + \frac{B}{2}\beta^2 (k_y^2 + k_z^2)$$

The effective mass is defined as: $m^* = \dfrac{\hbar^2}{\dfrac{d^2E}{dk^2}}$

Along (1 0 0) direction (longitudinal direction), the effective mass becomes:

$$m_l^* = \frac{\hbar^2}{\dfrac{d^2E}{dk_x^2}} = \frac{\hbar^2}{A\alpha^2}$$

Along the two transverse directions, the effective mass becomes:

$$m_t^* = \frac{\hbar^2}{\dfrac{d^2E}{dk_y^2}} = \frac{\hbar^2}{B\beta^2}$$

Finally, the density-of-states effective mass is given by:

$$D.O.S.\ m^* = (m_l^* m_t^{*2})^{1/3} = \frac{\hbar^2}{(AB^2\alpha^2\beta^4)^{1/3}}$$

Prob. 3.4

For the given band structure, find the temperature at which the number of electrons in the Γ minima and the X minima equal.

From Eq.(3-15), we have:

$$\frac{n_X}{n_\Gamma} = \frac{N_{cX}}{N_{c\Gamma}} e^{-\frac{0.35}{kT}}$$

Given that there are 6 X minima along the < 100 > directions, from Eq.(3-16b) we get:

$$\begin{cases} N_{cX} \propto 6 \cdot (0.30)^{3/2} \\ N_{c\Gamma} \propto (0.065)^{3/2} \end{cases}$$

$$\frac{n_X}{n_\Gamma} = 6 \times \left(\frac{0.30}{0.065}\right)^{\frac{3}{2}} e^{-\frac{0.35}{kT}}$$

When $n_\Gamma = n_X$, we obtain

$$e^{\frac{0.35}{kT}} = 6 \times \left(\frac{0.30}{0.065}\right)^{\frac{3}{2}}$$

That is: $kT = 0.0857\text{eV}$ or, $T = \mathbf{988K}$

Prob. 3.5

For the given band structure, calculate and sketch how the conductivity varies from low T to high T and find the ratio of the conductivities at 1000°C and 300°C.

$$n = n_\Gamma + n_L = n_\Gamma\left(1 + (15)^{3/2} e^{-\frac{0.30}{kT}}\right)$$

= constant, independent of temperature according to the problem

From Eq.(3-15), we have:

$$\frac{n_L}{n_\Gamma} = \frac{N_{cL}}{N_{c\Gamma}} e^{-\frac{0.30}{kT}}$$

$$n_\Gamma = N_{c\Gamma} e^{\frac{E_F - E_{c\Gamma}}{kT}}$$

$$n_L = N_{cL} e^{\frac{E_F - E_{c\Gamma}}{kT}} \cdot e^{\frac{-E_t}{kT}} = \left(\frac{15 m_\Gamma^*}{m_\Gamma^*}\right)^{3/2} n_\Gamma e^{\frac{-E_t}{kT}} = (15)^{3/2} n_\Gamma e^{\frac{-E_t}{kT}}$$

$$\sigma = q[n_\Gamma \mu_\Gamma + n_L \mu_L] = q[n_\Gamma \mu_\Gamma + n_L \frac{\mu_\Gamma}{50}]$$

$$= q n_\Gamma \mu_\Gamma \left[1 + \frac{(15)^{3/2}}{50} e^{-\frac{0.30}{kT}} \right]$$

$$= q \frac{n}{1 + (15)^{3/2} e^{-\frac{0.30}{kT}}} \mu_\Gamma \left[1 + \frac{(15)^{3/2}}{50} e^{-\frac{0.30}{kT}} \right]$$

$$= q n \mu_\Gamma \frac{1 + \frac{(15)^{3/2}}{50} e^{-\frac{0.30}{kT}}}{1 + (15)^{3/2} e^{-\frac{0.30}{kT}}}$$

$$= q n \mu_\Gamma \left[\frac{1 + \frac{58.1}{50} e^{-\frac{0.30}{kT}}}{1 + 58.1 \times e^{-\frac{0.30}{kT}}} \right]$$

$$T \ll \frac{E_s}{k},\ \sigma = \sigma_0 = q n \mu_\Gamma$$

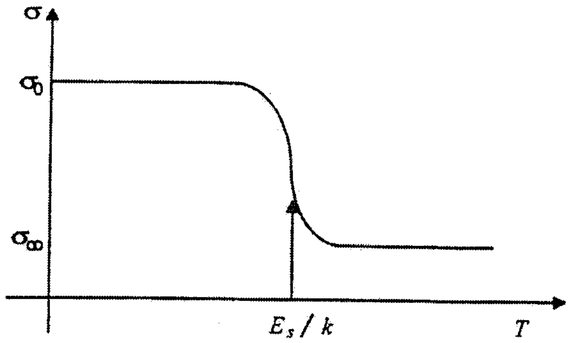
$$T \gg \frac{E_s}{k},\ \sigma = \sigma_\infty = q n \mu_\Gamma \left(\frac{1 + 1.16}{1 + 58.1} \right)$$

$$\frac{\sigma(1000°C)}{\sigma(300°C)} = \mathbf{0.254}$$

If the Γ to L separation is assumed to be 0.35 eV instead of 0.30 eV, we get:

$$\sigma = qn\mu_{\Gamma}\left[\frac{1+\frac{58.1}{50}e^{-\frac{0.35}{kT}}}{1+58.1\times e^{-\frac{0.35}{kT}}}\right]$$

$$\frac{\sigma\,(1000°C)}{\sigma\,(300°C)} = \mathbf{0.322}$$



Prob. 3.6

Find E_g for Si from Fig. 3-17.

$$\ln n_{i1} = \ln\sqrt{N_c N_v} - \frac{E_g}{2k}\left(\frac{1}{T_1}\right)$$

$$\ln n_{i2} = \ln\sqrt{N_c N_v} - \frac{E_g}{2k}\left(\frac{1}{T_2}\right)$$

$$\ln n_{i1} - \ln n_{i2} = \frac{E_g}{2k}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

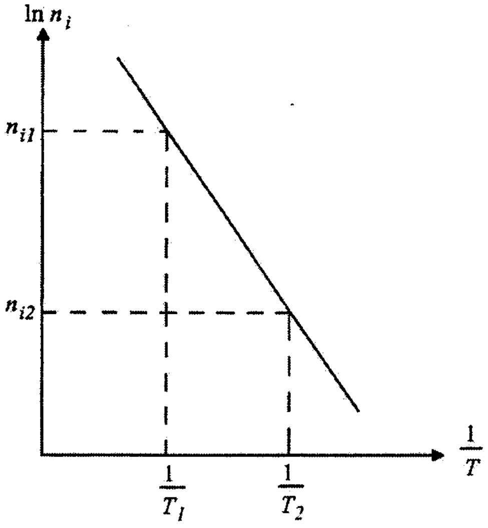
For Si, $n_{i2} = 10^8$ at $\frac{1}{T_2} = 4\times10^{-3}$

$n_{i1} = 3\times10^{14}$ *at* $\frac{1}{T_1} = 2\times10^{-3}$

$$E_g = 2k\ln(n_{i1}/n_{i2})/(\frac{1}{T_2} - \frac{1}{T_1})$$

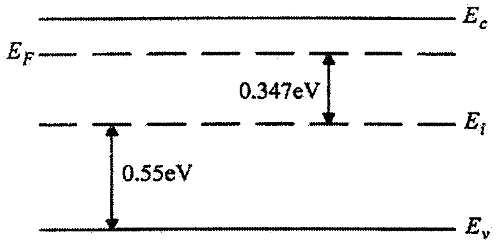
$$= \frac{2(8.62\times10^{-5})\cdot\ln(3\times10^{14}/10^8)}{(4-2)\cdot10^{-3}} = \mathbf{1.3\ eV}$$

This result is only approximate, since we neglect the temperature dependencies of N_C, N_V, and E_g.



Prob. 3.7

Show that Eq.(3-25) results from Eqs.(3-15) and (3-19) and find the position of the Fermi level relative to E_i at 300K



$$n_0 = N_c e^{-(E_c-E_F)/kT} = N_c e^{-(E_c-E_i)/kT} e^{(E_F-E_i)/kT}$$
$$= n_i e^{(E_F-E_i)/kT}$$
$$p_0 = n_i^2 / n_0 = n_i e^{(E_i-E_F)/kT}$$
$$n_0 = 10^{16} = 1.5\times10^{10} \times e^{(E_F-E_i)/0.0259}$$
$$E_F - E_i = 0.0259\times\ln(6.667\times10^5) = \mathbf{0.347eV}$$

Prob. 3.8

Find the displacement of E_i from the middle of E_g for Si.

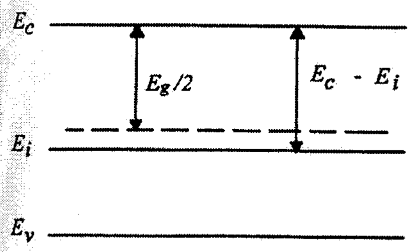
E_i is not exactly in the middle of the gap because the density of states N_C and N_V differ. Equating Eqs.(3-21) and (3-23).

$$N_c e^{-(E_c-E_i)/kT} = \sqrt{N_c N_v} e^{-Eg/2kT}$$
$$E_g/2-(E_c-E_i) = kT\ln(N_v/N_c)^{1/2} = kT\ln(m_p^*/m_n^*)^{3/4}$$

For Si at 300K,

$$E_g/2-(E_c-E_i) = 0.0259\times\frac{3}{4}\ \ln\ (0.56/1.1) = \mathbf{-0.013\ eV}$$

Thus, E_i is about $kT/2$ below the center of the gap.



Prob. 3.9

(a) Explain why holes appear at the top of the valence band.

Electron energy is plotted "up" in band diagrams such as Fig. 3-5. Thus conduction band electrons relax to the bottom of the conduction band. Holes, having positive charge, have energies which increase oppositely to that of negatively charged electrons. That is, hole energy would be plotted "down" on an electron energy diagram such as Fig.3-5. Holes therefore relax to the lowest hole energy available to them; i.e. the "top" of the valence band.

(b) Explain why Si doped with 10^{14} cm^{-3} donors is n-type at 400K, but Ge is not.

According Fig. 3-17, the intrinsic concentration n_i at 400K is

n_i(400K) ~ 10^{15} cm^{-3} for Ge
~ 10^{13} cm^{-3} for Si

Thus at this temperature, $N_d \gg n_i$ for Si, $N_d \ll n_i$ for Ge.

Prob. 3.10.

For Si with $N_d - N_a = 4 \times 10^{15}$ cm^{-3}, find E_F and R_H.

$$n_0 = N_d - N_a = 4\times10^{15} = n_i e^{(E_F - E_i)/kT}$$

$$E_F - E_i = kT\ln(n_0/n_i)$$
$$= 0.0259\ln(4\times10^{15}/1.5\times10^{10}) = \mathbf{0.324eV}$$

$$R_H = -(qn_0)^{-1} = -(1.6\times10^{-19}\times4\times10^{15})^{-1} = \mathbf{-1562.5\ cm^3/C}$$

Prob. 3.11.

(a) Find the value of n_0 for minimum conductivity.

$$\sigma = q(n\mu_n + p\mu_p) = q(n\mu_n + \mu_p n_i^2/n)$$

$$\frac{d\sigma}{dn} = q(\mu_n - \mu_p n_i^2/n^2)$$

Setting this equal to zero and defining n_m as the electron concentration for minimum conductivity, we have

$$n_m^2 = n_i^2 \mu_p/\mu_n, \; n_m = n_i\sqrt{\mu_p/\mu_n}$$

(b) What is $\sigma_{\min}$?

$$\sigma_{\min} = qn_i(\mu_n\sqrt{\mu_p/\mu_n} + \mu_p\sqrt{\mu_n/\mu_p}) = 2qn_i\sqrt{\mu_n\mu_p}$$

(c) Calculate $\sigma_{\min}$ and σ_i for Si

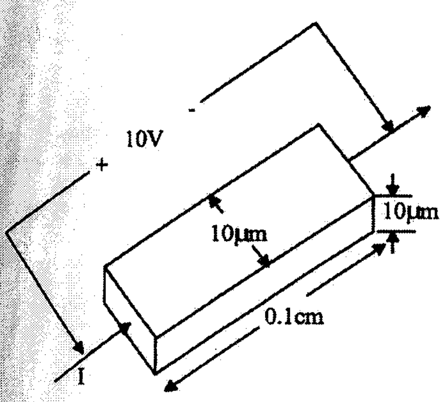
For Si,

$$\sigma_{\min} = 2(1.6\times10^{-19})(1.5\times10^{10})(1350\times480)^{1/2}$$
$$= 3.9\times10^{-6}(\Omega\cdot\text{cm})^{-1}$$
$$\sigma_i = qn_i(\mu_n + \mu_p) = (1.6\times10^{-19})(1.5\times10^{10})(1830)$$
$$= \mathbf{4.4\times10^{-6}(\Omega\cdot cm)^{-1}}$$

or, take the reciprocal of ρ_i in Appendix III.

Prob. 3.12

(a) A Si bar 0.1 cm long and 100 µm² in cross sectional area is doped with 10^{17} cm^{-3} antimony. Find the current at 300K with 10V applied.



From Fig.3-23, $\mu_n = 700$ cm²/V-s

$\sigma = q\mu_n n_0 = 1.6\times10^{-19}\times700\times10^{17} = 11.2\,(\Omega\cdot\text{cm})^{-1} = \rho^{-1}$

$\rho = 0.0893\,\Omega\cdot\text{cm}$

$R = \rho L/A = 0.0893\times0.1/10^{-6} = 8.93\times10^{3}\,\Omega$

$I = V/R = 10/(8.93\times10^{3}) = \mathbf{1.12\,mA}$

Repeat for a length of 1µm.

Now $\mathcal{E}$ = 10V/10^{-4} cm = 10^5 V/cm, which is in the velocity saturation regime. From Fig.3-24, $v_s = 10^7$ cm/s

$I = qAnv_s = (1.6\times10^{-19})(10^{-6})(10^{17})(10^{7}) = \mathbf{0.16\,A}$

(b) How long does it take an average electron to drift 1 µm in pure Si at an electric field of 100V/cm? Repeat for 10^5 V/cm.

From Appendix III, $\mu_n = 1350$ cm²/V-s

low field: $v_d = \mu_n\mathcal{E} = 1350 \times 100 = 1.35 \times 10^5$ cm/s

$t = L/v_d = 10^{-4}/(1.35 \times 10^5) = 7.4 \times 10^{-10}$ s = **0.74 ns**

high field: scattering-limited velocity $v_s = 10^7$ cm/s (Fig. 3-24)

$t = 10^{-4}/10^7 = 10^{-11}$ s = **10 ps**

Prob. 3.13

A perfect III-V semiconductor is doped with column VI and II impurities. For the given μ_n, μ_p, calculate the energy levels introduced in the bandgap.

$$\mu_n = \frac{qt}{m_n^*} \Rightarrow m_n^* = \frac{1.6\times10^{-19}\times10^{-13}}{1000(10^{-4}\frac{m^2}{cm^2})} m_0 = 1.6\times10^{-31}\ \text{kg} = 0.176 m_0$$

$$m_p^* = 1.408 m_0$$

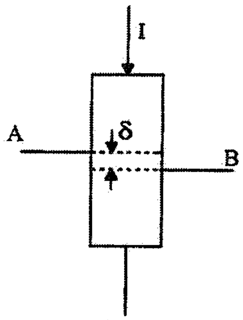
From Eq.(3 - 8) and using the value of the ground state energy of a H atom (1 Rydberg) = 13.6 eV :

$$E_D = \frac{13.6\frac{m_n^*}{m_0}}{\epsilon_r^2} = \mathbf{14.2meV}\ \text{below}\ E_C$$

$$E_A = \mathbf{113.6meV}\ \text{above}\ E_V$$

Prob. 3.14

Find V_H with Hall probes misaligned.



Displacement of the probes by an amount δ gives a small IR drop V_δ in addition to V_H. The Hall voltage reverses when $\mathscr{B}$ is reversed; however, V_δ is insensitive to the direction of the magnetic field. Thus,

$$\text{with } \mathscr{B} \text{ positive}: \ V_{AB}^+ = V_H + V_\delta$$

$$\text{with } \mathscr{B} \text{ negative}: \ V_{AB}^- = -V_H + V_\delta$$

$$\text{subtracting,} \qquad V_{AB}^+ - V_{AB}^- = 2V_H$$

We obtain the true Hall voltage from $V_H = \frac{1}{2}(V_{AB}^+ - V_{AB}^-)$.

Prob. 3.15

Find the position of the Fermi level for 11 electrons in an infinite 1-D potential well 100Å wide and the probability of exciting a carrier to the first excited state.

The energy levels are given by:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Since electrons obey the Pauli principle, only 2 electrons (opposite spin) are in each level.

Therefore, we can occupy up to $n = 6$ level $\left(\frac{11}{2} = 5 \text{ filled} + 1 \text{ half filled level}\right)$

$$E_6 = \frac{6^2\pi^2\hbar^2}{2mL^2}, \text{ where } m = \text{free electron mass, } L = 100\text{Å}$$

$$= 6^2(0.00120)\text{ eV} = 0.0432\text{ eV}$$

E_F is the highest filled level at 0K

Therefore, $E_F = E_6 = 0.0432$ eV.

First excited level is 7th level

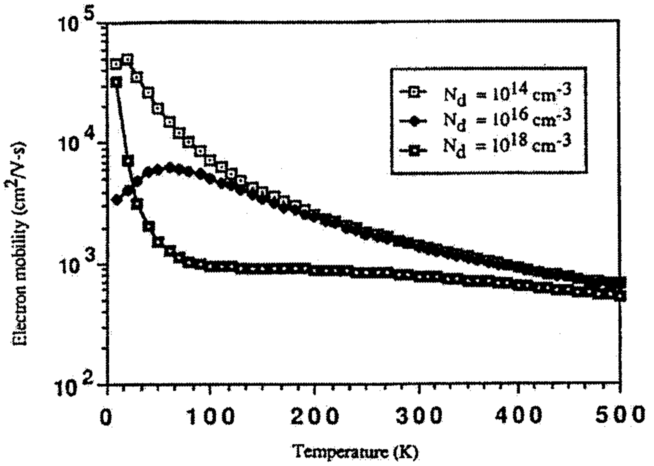
$$E_7 = \frac{7^2\pi^2\hbar^2}{2mL^2} = 0.0588\text{ eV}$$

Use Fermi - Dirac statistics for electrons :

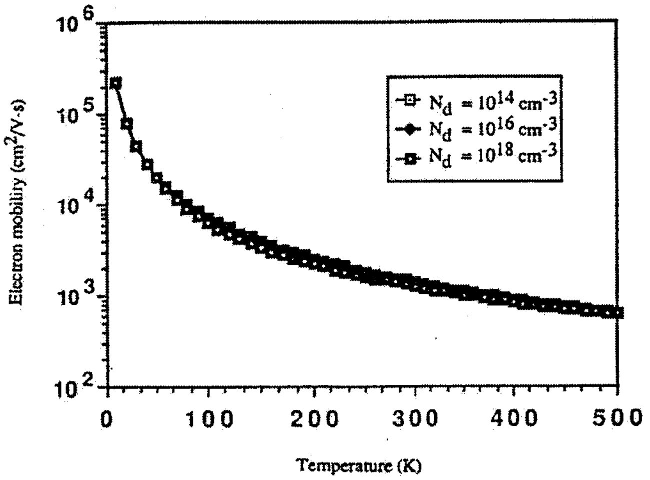
$$\text{Probability} = \frac{1}{1+e^{\frac{E_7-E_F}{kT}}} = \frac{1}{1+e^{\frac{E_7-E_6}{kT}}} = \frac{1}{1+e^{(0.0588-0.0432)/0.0259}}$$

At 300K, this is **0.354.**

Prob. 3.16



Prob. 3.17



When freeze-out occurs, ionized impurity scattering disappears, and only the phonon scattering remains. In real Si, other mechanisms, including neutral impurity scattering, contribute to mobility.

Prob. 3.18

Find the hole concentration and mobility with Hall measurement on a p-type semiconductor bar.

The voltage measured is the Hall voltage plus the ohmic drop. The sign of V_H changes with the magnetic field, but the ohmic voltage does not.

$$\text{True, } V_{Hall} = \frac{V_{H_1} - V_{H_2}}{2} = 3\,\text{mV}$$

Thus the ohmic drop is $3.2 - 3.0 = 0.2\,\text{mV}$

From Eq.(3 - 50)

$$p_0 = \frac{(3\times10^{-3}A)(10\times10^{-5}\text{Wb/cm}^2)}{q(20\times10^{-4}\text{cm})(3\times10^{-3}V)} = 3.125\times10^{17}\text{cm}^{-3}$$

$$\rho = \frac{\left(\dfrac{0.2\,\text{mV}}{3\,\text{mA}}\right)}{\dfrac{2\mu\text{m}}{500\mu\text{m}\times20\mu\text{m}}} = 0.033\,\Omega\cdot\text{cm} = \frac{1}{q\mu_p p_0}$$

$$\mu_p = \frac{1}{q\rho p_0} = \frac{1}{1.6\times10^{-19}(0.033)(3.125\times10^{17})} = \mathbf{600\,cm^2/(V\cdot s)}$$

Prob. 3.19

Calculate the conductivity of a hypothetical semiconductor at 600K.

The intrinsic conductivity is given as

$$\sigma_i = qn_i(\mu_n + \mu_p) = q\left(\sqrt{N_cN_v}\,e^{-\frac{E_g}{2kT}}\right)\times2000 = 4\times10^{-6}(\Omega\cdot\text{cm})^{-1}$$

$$4\times10^{-6} = 1.6\times10^{-19}(10^{19})(2000)e^{-\frac{E_g}{2kT}}$$

As T goes from 300K to 600K, E_g, N_c, N_v do not change, and

$e^{-\frac{E_g}{2kT_1}}$ increases to $e^{-\frac{E_g}{2k(2T_1)}}$, where $T_1 = 300K$.

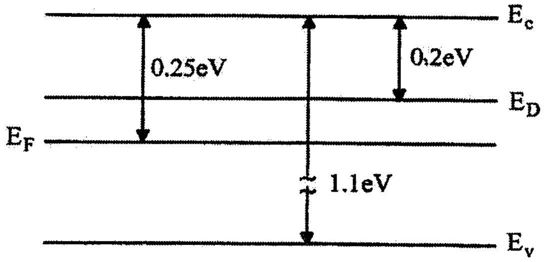
Therefore,

$$e^{-\frac{E_g}{2k(2T_1)}} = \sqrt{\frac{4\times10^{-6}}{1.6\times10^{-19}\times10^{19}\times2000}} = 3.54\times10^{-5}$$

$$\sigma = \frac{\sigma_i}{3.54\times10^{-5}} = \mathbf{0.113(\Omega\cdot cm)^{-1}}$$

Prob. 3.20

Calculate the number of electrons, holes, and n_i in the unknown semiconductor with E_F 0.25eV below E_c.



Incomplete ionization :

$$f(E_d)=\frac{1}{1+e^{\frac{0.05}{0.0259}}}=0.1267$$

$$n=(1-f)N_d=8.733\times10^{14}\text{cm}^{-3}$$

Also, $n=N_ce^{-\frac{E_c-E_F}{kT}}$

$$N_c=ne^{-\frac{E_c-E_F}{kT}}=8.733\times10^{14}\times e^{0.25/0.0259}$$

$$=1.359\times10^{19}\text{cm}^{-3}=N_v$$

$$p=N_ve^{-\frac{E_F-E_v}{kT}}=1.359\times10^{19}\times e^{-(1.1-0.25)/0.0259}=7.591\times10^{4}\text{cm}^{-3}$$

$$n_i=\sqrt{np}=\mathbf{8.142\times10^{9}cm^{-3}}$$

Prob. 3.21

Referring to Fig. 3.25, find the type, concentration and mobility of the majority carrier.

Given,

$B_z=10^{-4}\text{Wb/cm}^2$

From the sign of V_{AB}, we can see the majority carriers are electrons.

$$n_0=\frac{I_xB_z}{qt(-V_{AB})}=\frac{(10^{-3})(10^{-4})}{1.6\times10^{-19}(10^{-3})(2\times10^{-3})}=3.125\times10^{17}\text{cm}^{-3}$$

$$\rho=\frac{R}{L/wt}=\frac{V_{CD}/I_x}{L/wt}=\frac{0.1/10^{-3}}{0.5/0.01\times10^{-3}}=0.002\,\Omega\cdot\text{cm}$$

$$\mu_n=\frac{1}{\rho qn_0}=\frac{1}{(0.002)(1.6\times10^{-19})(3.125\times10^{17})}=\mathbf{10{,}000\ cm^2(V\cdot s)^{-1}}$$