Chapter 4

Prob. 4.1

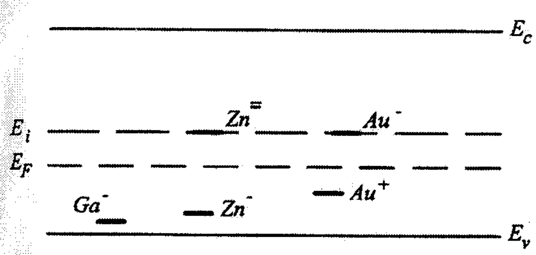
With E_F located 0.4 eV above the valence band in a Si sample, what charge state would you expect for most Ga, Zn, Au atoms in the sample?

From Fig.4-9, we have

Ga^-: $E_F >> Ga^-$: singly negative

Zn^-: $E_F > Zn^-$ but below $Zn^{=}$: singly negative

Au^0: $Au^+ < E_F < Au^-$: neutral.



Prob. 4.2

How much Zn must be added to exactly compensate a Si sample doped with 10^{16} cm^{-3} Sb?

$E_i = E_F \approx E_{Zn^=}$

$$\left.\begin{array}{l}\text{All } Zn^- \text{ state are filled} \\ \frac{1}{2}\text{ of } Zn^= \text{ states are filled}\end{array}\right\} \frac{3}{2} N_{Zn} \text{ electrons on Zn atoms}$$

For compensation :

$$\frac{3}{2}N_{Zn} = N_d \Rightarrow N_{Zn} = \frac{2}{3}N_d = \mathbf{0.667 \times 10^{16} cm^{-3}}$$

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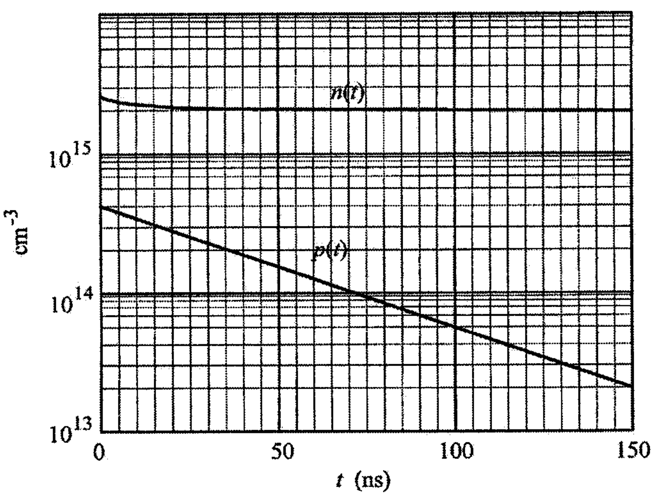
Prob. 4.3
Draw a semilogarithmic plot such as Fig. 4-7 for the given GaAs.

For the given GaAs,
$n_0 = 2 \times 10^{15}$ cm^{-3}, $p_0 = n_i^2/n_0 = 0.002$ (negligible)
At $t = 0$, $\Delta n = \Delta p = 4 \times 10^{14}$ cm^{-3}.



Prob. 4.4
Calculate the recombination coefficient for the low-level excitation in Prob. 4.3. Find the steady state excess carrier concentration.

$\alpha_r = 1/(\tau n_0) = [50 \times 10^{-9} \times 2 \times 10^{15}]^{-1} = 10^{-8}$
From Eq. (4-12), $g_{op} = \alpha_r[n_0\delta n + \delta n^2] = 10^{-8}[2 \times 10^{15}\,\delta n + \delta n^2]$
$\delta n^2 + 2 \times 10^{15}\,\delta n - 10^{28} = 0$ and $\delta n \sim 5 \times 10^{12}$ cm^{-3} $= \Delta n$
or, since the low-level lifetime is valid, $\Delta n = g_{op}\tau = 5 \times 10^{12}$ cm^{-3}.

Prob. 4.5

If $n_0 = Gx$, find $\mathcal{E}(x)$ for $n_0 >> n_i$. We also assume E_F remains below E_c.

At equilibrium:

$$J_n = q\mu_n n\mathcal{E} + qD_n \frac{dn}{dx} = 0$$

$$\mathcal{E}(x) = -\frac{D_n}{\mu_n}\frac{dn/dx}{n}$$

$$= -\frac{kT}{q}\frac{G}{Gx} = -\frac{kT}{q}x^{-1}$$

Prob. 4.6

Find the separation of the quasi-Fermi levels and the change of conductivity upon shining light on a Si sample.

The light induced electron-hole pair concentration is determined by:

$$\delta n = \delta p = g_{op}\tau = (10^{19})(10^{-5}) = 10^{14}\,\text{cm}^{-3}$$

$$<< \text{dopant concentration of } 10^{15}\,\text{cm}^{-3} \quad \Rightarrow \text{ low level}$$

$$n = 10^{15} + 10^{14} = 1.1\times 10^{15}\,\text{cm}^{-3}$$

$$p_0 + \delta p = \frac{n_i^2}{n_0} + \delta p = \frac{(1.5\times 10^{10})^2}{10^{15}} + 10^{14} \approx 10^{14}\,\text{cm}^{-3}$$

$$F_n - F_p = kT\ln\left(\frac{np}{n_i^2}\right) = 0.0259\ln\left(\frac{1.1\times 10^{29}}{2.25\times 10^{20}}\right) = 0.518\,\text{eV}$$

$$\mu_n = 1300\,\text{cm}^2/(\text{V}\cdot\text{s}) \text{ from Fig. 3.23}$$

$$\mu_p = \frac{D_p}{kT/q} = \frac{12}{0.0259} = 463\,\text{cm}^2/(\text{V}\cdot\text{s})$$

$$\Delta\sigma = q(\mu_n\delta n + \mu_p\delta p) = 1.6\times 10^{-19}(1300 + 463)(10^{14})$$

$$= \mathbf{0.0282\ (\Omega\cdot cm)^{-1}}$$

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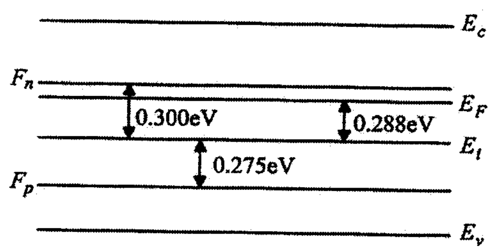
Prob. 4.7

Calculate the separation in the quasi-Fermi levels and draw a band diagram for an n-type Si being steadily illuminated.



The induced electron concentration is

$\delta n = g_{op}\tau = (10^{21})(10^{-6}) = 10^{15}\text{cm}^{-3}$

which is comparable with $N_d = 10^{15}\text{cm}^{-3}$.

$\Rightarrow$ this is not low level and δn^2 cannot be neglected.

$$g_{op} = \alpha_r n_0 \delta n + \alpha_r \delta n^2$$

$$\alpha_r = \frac{1}{\tau_n n_0} = \frac{1}{(10^{-6})(10^{15})} = 10^{-9}\text{cm}^3 s^{-1}$$

$$\Rightarrow 10^{21} = (10^{-9})(10^{15})\delta n + 10^{-9}\delta n^2$$

Solve for δn $\Rightarrow$ $\delta n = 6.18\times10^{14}\text{cm}^{-3} = \delta p$

$$F_n - E_i = kT\ln\left(\frac{n_0+\delta n}{n_i}\right) = 0.0259\ln\left(\frac{10^{15}+6.18\times10^{14}}{1.5\times10^{10}}\right) = \mathbf{0.300eV}$$

$$E_i - F_p = kT\ln\left(\frac{\delta n}{n_i}\right) = 0.0259\ln\left(\frac{6.18\times10^{14}}{1.5\times10^{10}}\right) = \mathbf{0.275eV}$$

$$E_F - E_i = kT\ln\left(\frac{n_0}{n_i}\right) = 0.0259\ln\left(\frac{10^{15}}{1.5\times10^{10}}\right) = \mathbf{0.288eV}$$

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Prob. 4.8

Calculate the current in a long Si bar as described.

$$\alpha_r = \frac{1}{\tau n_0} = \frac{1}{(10^{-4})(10^{16})} = 10^{-12} cm^3 s^{-1}$$

$$g_{op} = \alpha_r n_0 \delta n + \alpha_r \delta n^2 \quad \Rightarrow$$

$$10^{20} = 10^{-12}[(10^{16})\delta n + \delta n^2] \quad \Rightarrow$$

$$10^{-32}\delta n^2 + 10^{-16}\delta n - 1 = 0$$

$$(\overline{\delta n})^2 + \overline{\delta n} - 1 = 0, \text{ where } \overline{\delta n} = 10^{-16}\delta n$$

Solve for $\overline{\delta n}$ to get $\delta n \quad \Rightarrow$

$$\delta n = 10^{16}\frac{-1+\sqrt{1+4}}{2}$$

$$\delta n = 6.18 \times 10^{15} cm^{-3} = \delta p$$

Assume the α_r is the low level, even if the calculation may require high level injection assumption.

No light:

$$\mathcal{E} = \frac{10\text{V}}{2\text{cm}} = 5\text{V/cm}$$

$\mu_n = 1070\ \text{cm}^2/(\text{V}\cdot\text{s})$ from Fig. 3.23, $\mu_p = 500\ \text{cm}^2/(\text{V}\cdot\text{s})$

$$I = Aqn_0\mu_n\mathcal{E} = (0.05)(1.6\times10^{-19}\times10^{16}\times1070\times5) = \mathbf{0.428A}$$

With light:

$$I = A\cdot\left(q\{(n_0+\delta n)\mu_n + \delta p\mu_p\}\right)\cdot\mathcal{E}$$

$$= 0.05(1.6\times10^{-19}(1.73\times10^{19} + 3.09\times10^{18}))\cdot 5$$

$$= \mathbf{0.816A}$$

High field + light:

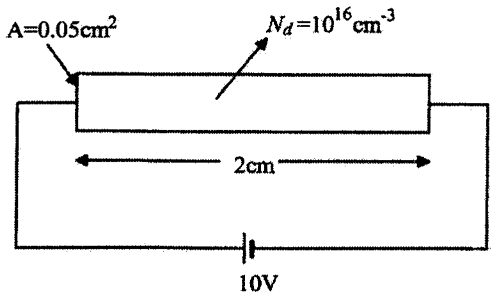
For electrons, saturation velocity $v_s = 10^7$ cm/s

For holes, assume μ_p is same as for low field, $\mathcal{E} = \frac{100{,}000}{2} = 50{,}000\text{V/cm}$

$$I = Aq[(n_0+\delta n)v_s + \delta p\mu_p\mathcal{E}]$$

$$= (0.05)(1.6\times10^{-19})(1.618\times10^{16}\times10^7 + 6.18\times10^{15}\times500\times5\times10^4)$$

$$= \mathbf{2.53\times10^3\ A}$$



Prob. 4.9

Design a 5-μm CdS photoconductor with 10 MΩ dark resistance, 0.5 cm square. Assume $\tau = 10^{-6}$ *s and* $N_d = 10^{14}$ cm^{-3}.

In the dark, $\sigma = q\mu_n n_0$, neglecting p_0

$\rho = \sigma^{-1} = [1.6 \times 10^{-19} \times 250 \times 10^{14}]^{-1} = 250\ \Omega\text{-cm}$
$R = 10^7\ \Omega = \rho L/wt$, thus $L = 10^7\ (5\times10^{-4})w/250$

Since this is a design problem, there are many solutions. For example, choosing w = 0.5mm, L = 1 cm:
with the light on, $g_{op} = 10^{21}$ EHP/cm^3-s

$$\sigma = q[(n_0 + \Delta n)\mu_n + \Delta p\mu_p]$$
$$= 1.6\times10^{-19}[(1.1\times10^{15})250 + 10^{15}\times15]$$
$$= 4.64\times10^{-2}(\Omega\cdot\text{cm})^{-1}$$
$$R = \frac{\rho L}{wt} = [(4.64\times10^{-2})(0.05)(5\times10^{-4})]^{-1}$$
$$R = 8.62\times10^5\Omega$$
$$\Delta R = 10^7 - 8.62\times10^5 = \mathbf{9.14M\Omega}$$

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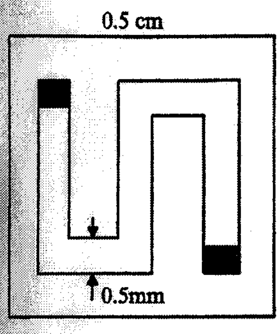


Prob. 4.10

Calculate the steady state separation between F_P and E_C at $x = 1000\text{Å}$ in a very long p-type Si bar with steady state excess hole concentration. Also find the hole current there and the excess stored hole charge.

$$D_p = \frac{kT}{q}\mu_p = 0.0259 \times 500 = 12.95\,\text{cm}^2/\text{s}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{12.95 \times 10^{-10}} = 3.6 \times 10^{-5}\,\text{cm}$$

$$p = p_0 + \Delta p e^{-\frac{x}{L_p}} = 10^{17} + 5 \times 10^{16} e^{-\frac{10^{-5}}{3.6\times10^{-5}}}$$

$$= 1.379 \times 10^{17} = n_i e^{(E_i - F_p)/kT} = (1.5 \times 10^{10}\,\text{cm}^{-3}) e^{(E_i - F_p)/kT}$$

$$E_i - F_p = \left(\ln \frac{1.379 \times 10^{17}}{1.5 \times 10^{10}} \right) \cdot 0.0259 = 0.415\text{eV}$$

$$E_c - F_p = 1.1/2\text{eV} + 0.415\text{eV} = \mathbf{0.965eV}$$

Hole current :

$$I_p = -qAD_p \frac{dp}{dx} = qA\frac{D_p}{L_p}(\Delta p)e^{-\frac{x}{L_p}}$$

$$= 1.6\times10^{-19}\times0.5\times\frac{12.95}{3.6\times10^{-5}}\times5\times10^{16}e^{-\frac{10^{-5}}{3.6\times10^{-5}}}$$

$$= \mathbf{1.09\times10^{3}\,A}$$

$$Q_p = qA(\Delta p)L_p$$

$$= 1.6\times10^{-19}(0.5)(5\times10^{16})(3.6\times10^{-5})$$

$$= \mathbf{1.44\times10^{-7}\,C}$$

Prob. 4.11

Find the photocurrent ΔI in terms of τ_n and τ_t for a sample dominated by μ_n.

$$\Delta\sigma \approx q\mu_n\Delta n = q\mu_n g_{op}\tau_n$$

$$\Delta I = V/\Delta R = VA\Delta\sigma / L = VAq\mu_n g_{op}\tau_n/L$$

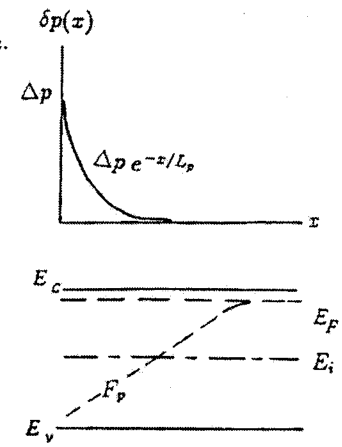
The transit time is

$$\tau_t = \frac{L}{v_d} = \frac{L}{\mu_n V/L} = \frac{L^2}{\mu_n V}$$

$$\Delta I = qALg_{op}\tau_n/\tau_t$$

Prob. 4.12

Find $F_p(x)$ for an exponential excess hole distribution.



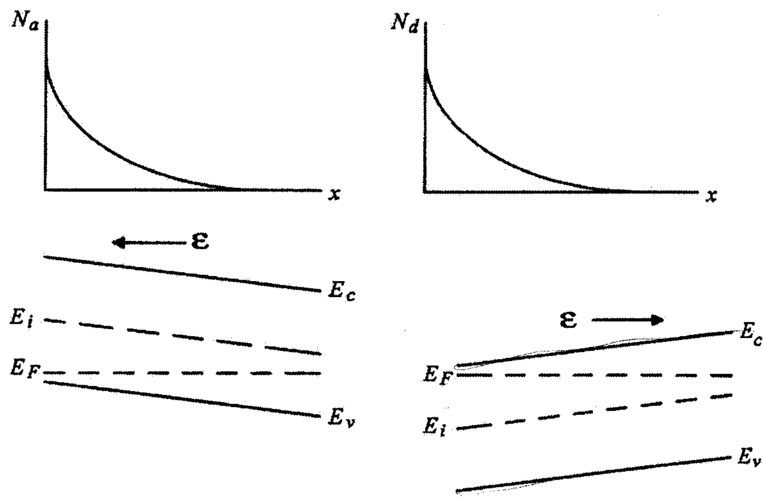
For $\delta p \gg p_0$,

$$p(x) \simeq \delta p(x) = \Delta p\, e^{-x/L_p}$$
$$= n_i\, e^{(E_i - F_p)/kT}$$
$$E_i - F_p = kT \ln \frac{\delta p}{n_i}$$
$$= kT \ln \frac{\Delta p}{n_i} e^{-x/L_p}$$
$$= kT \left[\ln \frac{\Delta p}{n_i} - \frac{x}{L_p} \right]$$

We assume the excess minority hole concentration is small compared to n_0 throughout, so no band bending is observable on this scale.

Prob. 4.13

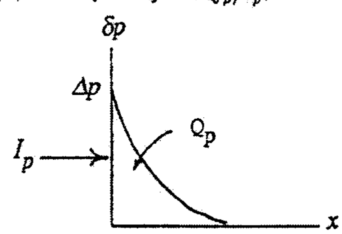
Sketch the equilibrium bands and field in an exponential acceptor distribution. Repeat for donors.



Prob. 4.14

Show the hole current feeding an exponential $\delta p(x)$ can be found from Q_p/τ_p.

From Fig. 4-17,



$$Q_p = qA\int_0^\infty \Delta p\, e^{-x/L_p}dx$$
$$= qAL_p\Delta p$$
$$I_p = \frac{Q_p}{\tau_p} = qAL_p\Delta p/\tau_p = qAD_p\Delta p/L_p$$

The charge distribution Q_p disappears by recombination and must be replaced by injection on the average every τ_p seconds. Thus the current injected is Q_p/τ_p.

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Prob. 4.15

Include recombination in the Haynes-Shockley experiment and find τ_p if the peak is 4 times as large for $t_d = 50$ μs as it is for 200 μs.

To include recombination, let the peak value vary as $exp(-t/\tau_p)$

$$\delta p(x,t) = \frac{\Delta p e^{-t/\tau_p}}{\sqrt{4\pi D_p t}} \exp(-x^2/4D_p t)$$

At the peak ($x = 0$),

$$V_p = \text{peak} = B\frac{\Delta p e^{-t/\tau_p}}{\sqrt{4\pi D_p t}}, \text{ where B is a proportionality constant.}$$

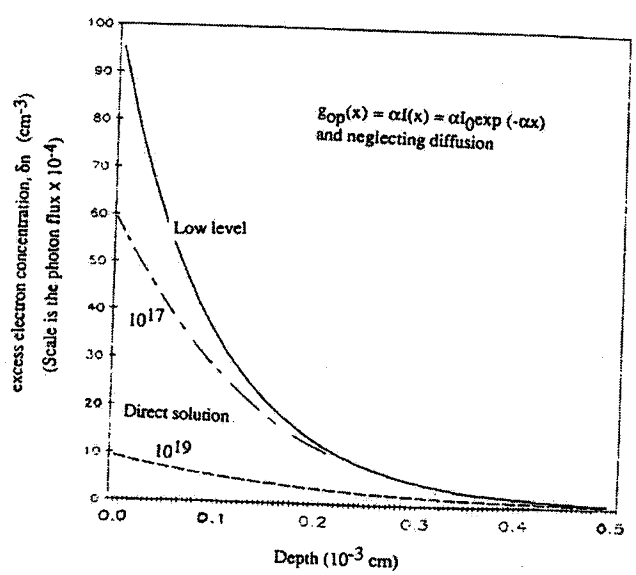
$$\frac{V_{p1}}{V_{p2}} = \sqrt{\frac{t_2}{t_1}}\frac{e^{-t_1/\tau_p}}{e^{-t_2/\tau_p}} = \sqrt{\frac{t_2}{t_1}}e^{(t_2-t_1)/\tau_p}$$

$$\frac{80}{20} = \sqrt{\frac{200}{50}}e^{150/\tau_p}$$

$$\frac{150}{\tau_p} = \ln\frac{4}{\sqrt{4}}$$

$$\tau_p = \frac{150}{\ln 2} = \mathbf{216.4\mu s}$$

Prob. 4.16



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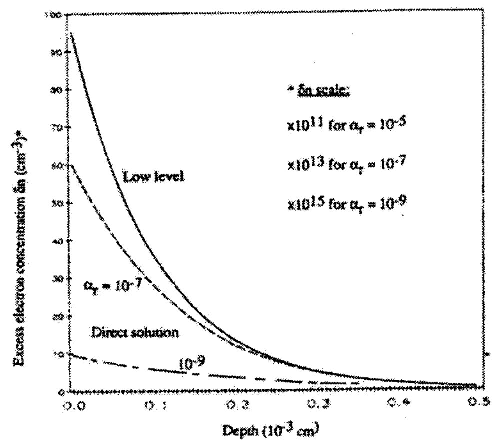
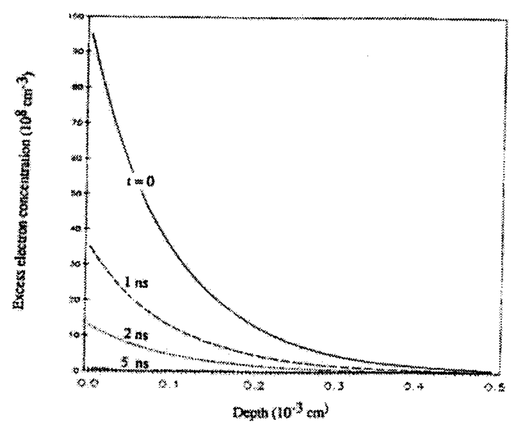
Prob. 4.17



Prob. 4.18



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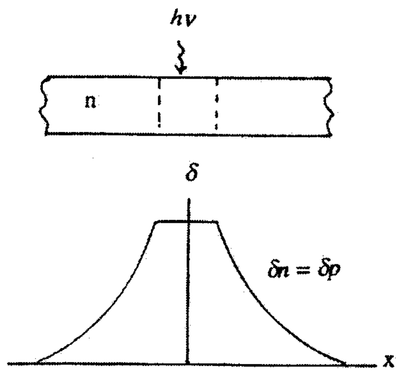
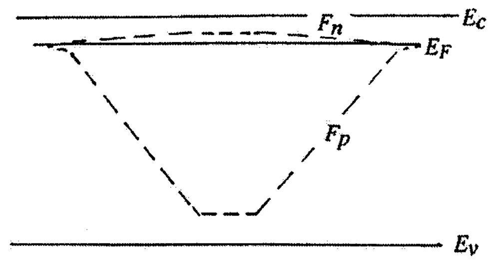
Prob. 4.19

Sketch the quasi-Fermi levels in an n-type sample illuminated in a narrow region.



Excess carriers diffuse and recombine, decaying exponentially away from the illuminated region



As in prob. 4.12, the quasi-Fermi levels vary linearly outside the excitation region while $\delta p \gg p_0$

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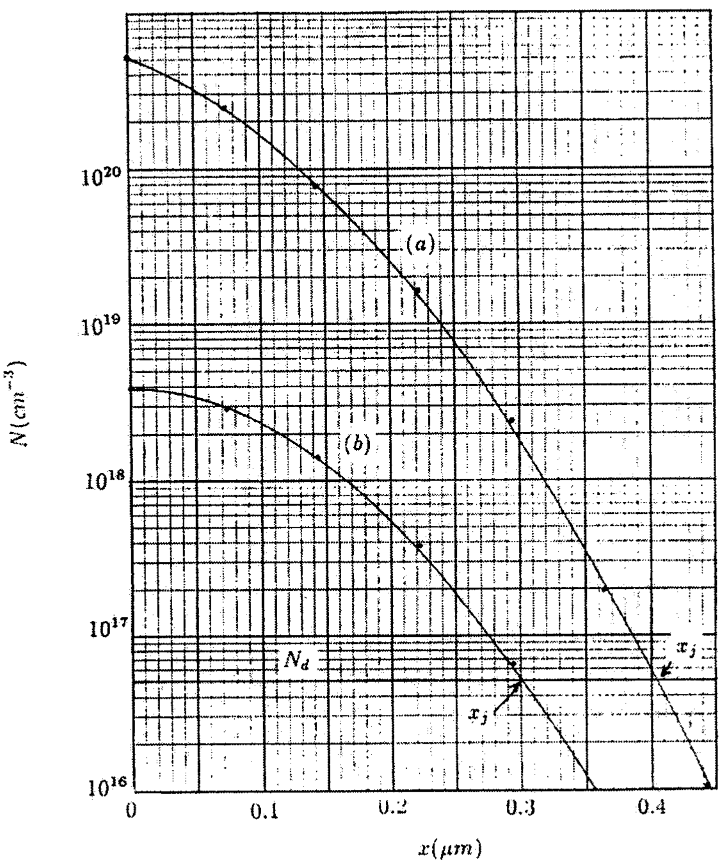
(b) After diffusion. $N_0 = N_s/\sqrt{\pi Dt} = \frac{5 \times 10^{13}}{0.1302 \times 10^{-4}} = 3.84\times10^{18}$

$x(\mu m)$	u	$\exp(-u^2)$	$N(x)$
0.0735	0.5	0.78	3.0×10^{18}
0.1470	1.0	0.37	1.4×10^{18}
0.2205	1.5	0.105	4.0×10^{17}
0.2940	2.0	0.018	6.9×10^{16}
0.3675	2.5	0.0019	7.3×10^{15}

$x_j = 0.3\ \mu m.$



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